10.6 Exercises

VOCABULARY CHECK: Fill in the blanks.

1. If f and g are continuous functions of t on an interval I, the set of ordered pairs $(f(t), g(t))$ is a ________ ________ C. The equations $x = f(t)$ and $y = g(t)$ are ________ equations for C, and t is the ________.
2. The ________ of a curve is the direction in which the curve is traced out for increasing values of the parameter.
3. The process of converting a set of parametric equations to a corresponding rectangular equation is called ________ the ________.

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

1. Consider the parametric equations $x = \sqrt{t}$ and $y = 3 - t$.
   (a) Create a table of x- and y-values using $t = 0, 1, 2, 3,$ and 4.
   (b) Plot the points (x, y) generated in part (a), and sketch a graph of the parametric equations.
   (c) Find the rectangular equation by eliminating the parameter. Sketch its graph. How do the graphs differ?
2. Consider the parametric equations $x = 4\cos^2\theta$ and $y = 2\sin\theta$.
   (a) Create a table of x- and y-values using $\theta = -\pi/2, -\pi/4, 0, \pi/4,$ and $\pi/2$.
   (b) Plot the points (x, y) generated in part (a), and sketch a graph of the parametric equations.
   (c) Find the rectangular equation by eliminating the parameter. Sketch its graph. How do the graphs differ?

In Exercises 3–22, (a) sketch the curve represented by the parametric equations (indicate the orientation of the curve) and (b) eliminate the parameter and write the corresponding rectangular equation whose graph represents the curve. Adjust the domain of the resulting rectangular equation if necessary.

3. $x = 3t - 3$, $y = 2t + 1$
4. $x = 3 - 2t$, $y = 2 + 3t$
5. $x = \frac{1}{4}t$, $y = t^2$
6. $x = t$, $y = t^3$
7. $x = t + 2$, $y = t^2$
8. $x = \sqrt{t}$, $y = 1 - t$
9. $x = t + 1$, $y = \dfrac{t}{t + 1}$
10. $x = t - 1$, $y = \dfrac{t}{t - 1}$
11. $x = 2(t + 1)$, $y = |t - 2|$
12. $x = |t - 1|$, $y = t + 2$
13. $x = 3\cos\theta$, $y = 3\sin\theta$
14. $x = 2\cos\theta$, $y = 3\sin\theta$
15. $x = 4\sin 2\theta$, $y = 2\cos 2\theta$
16. $x = \cos\theta$, $y = 2\sin 2\theta$
17. $x = 4 + 2\cos\theta$, $y = -1 + \sin\theta$
18. $x = 4 + 2\cos\theta$, $y = 2 + 3\sin\theta$
19. $x = e^{-t}$, $y = e^{3t}$
20. $x = e^{2t}$, $y = e^{t}$
21. $x = t^3$, $y = 3\ln t$
22. $x = \ln 2t$, $y = 2t^2$

In Exercises 23 and 24, determine how the plane curves differ from each other.

23. (a) $x = t$, $y = 2t + 1$ (b) $x = \cos\theta$, $y = 2\cos\theta + 1$
    (c) $x = e^{-t}$, $y = 2e^{-t} + 1$ (d) $x = e^{t}$, $y = 2e^{t} + 1$
24. (a) $x = t$, $y = t^2 - 1$ (b) $x = t^2$, $y = t^4 - 1$
    (c) $x = \sin t$, $y = \sin^2 t - 1$ (d) $x = e^{t}$, $y = e^{2t} - 1$

In Exercises 25–28, eliminate the parameter and obtain the standard form of the rectangular equation.

25. Line through (x_1, y_1) and (x_2, y_2): $x = x_1 + t(x_2 - x_1)$, $y = y_1 + t(y_2 - y_1)$
26. Circle: $x = h + r\cos\theta$, $y = k + r\sin\theta$
27. Ellipse: $x = h + a\cos\theta$, $y = k + b\sin\theta$
28. Hyperbola: $x = h + a\sec\theta$, $y = k + b\tan\theta$

In Exercises 29–36, use the results of Exercises 25–28 to find a set of parametric equations for the line or conic.

29. Line: passes through $(0, 0)$ and $(6, -3)$
30. Line: passes through $(2, 3)$ and $(6, -3)$
31. Circle: center: $(3, 2)$; radius: 4

32. Circle: center: $(-3, 2)$; radius: 5

33. Ellipse: vertices: $(\pm 4, 0)$; foci: $(\pm 3, 0)$

34. Ellipse: vertices: $(4, 7)$, $(4, -3)$;
foci: $(4, 5)$, $(4, -1)$

35. Hyperbola: vertices: $(\pm 4, 0)$; foci: $(\pm 5, 0)$

36. Hyperbola: vertices: $(\pm 2, 0)$; foci: $(\pm 4, 0)$

In Exercises 37–44, find a set of parametric equations for the rectangular equation using (a) $t = x$ and (b) $t = 2 - x$.

37. $y = 3x - 2$

38. $x = 3y - 2$

39. $y = x^2$

40. $y = x^3$

41. $y = x^2 + 1$

42. $y = 2 - x$

43. $y = \dfrac{1}{x}$

44. $y = \dfrac{1}{2x}$

In Exercises 45–52, use a graphing utility to graph the curve represented by the parametric equations.

45. Cycloid: $x = 4(\theta - \sin\theta)$, $y = 4(1 - \cos\theta)$

46. Cycloid: $x = \theta + \sin\theta$, $y = 1 - \cos\theta$

47. Prolate cycloid: $x = \theta - \frac{3}{2}\sin\theta$, $y = 1 - \frac{3}{2}\cos\theta$

48. Prolate cycloid: $x = 2\theta - 4\sin\theta$, $y = 2 - 4\cos\theta$

49. Hypocycloid: $x = 3\cos^3\theta$, $y = 3\sin^3\theta$

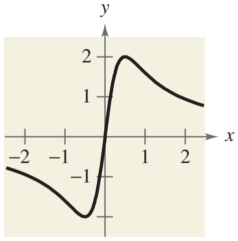
50. Curtate cycloid: $x = 8\theta - 4\sin\theta$, $y = 8 - 4\cos\theta$

51. Witch of Agnesi: $x = 2\cot\theta$, $y = 2\sin^2\theta$

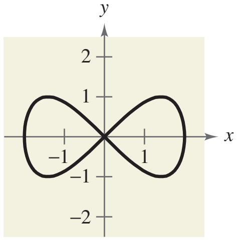
52. Folium of Descartes: $x = \dfrac{3t}{1 + t^3}$, $y = \dfrac{3t^2}{1 + t^3}$

In Exercises 53–56, match the parametric equations with the correct graph and describe the domain and range. [The graphs are labeled (a), (b), (c), and (d).]

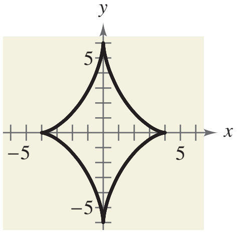
(a)



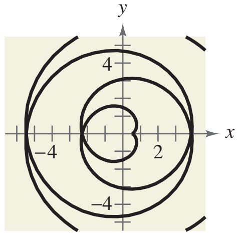
(b)



(c)



(d)



53. Lissajous curve: $x = 2\cos\theta$, $y = \sin 2\theta$

54. Evolute of ellipse: $x = 4\cos^3\theta$, $y = 6\sin^3\theta$

55. Involute of circle: $x = \frac{1}{2}(\cos\theta + \theta\sin\theta)$
$y = \frac{1}{2}(\sin\theta - \theta\cos\theta)$

56. Serpentine curve: $x = \frac{1}{2}\cot\theta$, $y = 4\sin\theta\cos\theta$

Projectile Motion **A projectile is launched at a height of h feet above the ground at an angle of θ with the horizontal. The initial velocity is v_0 feet per second and the path of the projectile is modeled by the parametric equations**

$$x = (v_0 \cos\theta)t \quad \text{and} \quad y = h + (v_0 \sin\theta)t - 16t^2.$$

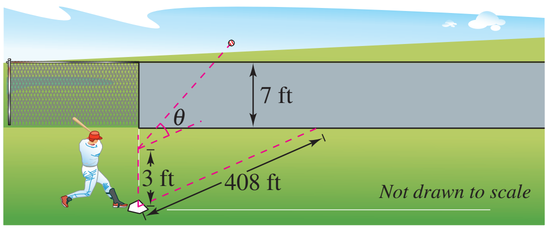
In Exercises 57 and 58, use a graphing utility to graph the paths of a projectile launched from ground level at each value of θ and v_0. For each case, use the graph to approximate the maximum height and the range of the projectile.

57. (a) $\theta = 60°$, $v_0 = 88$ feet per second
(b) $\theta = 60°$, $v_0 = 132$ feet per second
(c) $\theta = 45°$, $v_0 = 88$ feet per second
(d) $\theta = 45°$, $v_0 = 132$ feet per second

58. (a) $\theta = 15°$, $v_0 = 60$ feet per second
(b) $\theta = 15°$, $v_0 = 100$ feet per second
(c) $\theta = 30°$, $v_0 = 60$ feet per second
(d) $\theta = 30°$, $v_0 = 100$ feet per second

Model It

59. ***Sports*** The center field fence in Yankee Stadium is 7 feet high and 408 feet from home plate. A baseball is hit at a point 3 feet above the ground. It leaves the bat at an angle of θ degrees with the horizontal at a speed of 100 miles per hour (see figure).



(a) Write a set of parametric equations that model the path of the baseball.

(b) Use a graphing utility to graph the path of the baseball when $\theta = 15°$. Is the hit a home run?

(c) Use a graphing utility to graph the path of the baseball when $\theta = 23°$. Is the hit a home run?

(d) Find the minimum angle required for the hit to be a home run.